Precalc Warm Up #10-4

1. 5, 3, 1, -1, ... find the 10th term

2. 5, 1, 0.2, ... find the 10th term
   (give the exact answer)

3. $-1 + 3 + 7 + 11 + \ldots + 71 = ?$

HW Questions: p. 255

1. Find the common ratio, the 5th term and the general term of the following sequences

(a) 3, 6, 12, 24, . . .

(c) $2, \frac{2}{5}, \frac{2}{25}, \frac{2}{125}, \ldots$

(d) –1, 4, –16, 64, …

(f) $a^2, ab, b^2, \ldots$

2. Find the value(s) of x if each of the following are in geometric sequence

(a) $3, x, 48$

(b) $\frac{5}{2}, x, \frac{1}{2}$

a) $r = \frac{x}{3}$ $\quad r = \frac{48}{x}$

b) $\frac{x}{\frac{5}{2}} = \frac{\frac{1}{2}}{x}$ $\quad$ using common ratios =

$x^2 = \frac{1}{2} \cdot \frac{5}{2}$

$x = \pm \frac{\sqrt{5}}{2}$

3. The third and seventh terms of a geometric sequence are $\frac{3}{4}$ and 12 respectively

(a) Find the 10th term. → $g_{10} = g_1 (\pm 2)^9$

(b) What term is equal to 3072?

a)

$$\begin{array}{c|cc} n & 3 & 7 \\ \hline g_n & \frac{3}{4} & 12 \end{array}$$

4 jumps of r $\quad$ so: $\frac{3}{4}r^4 = 12$

$r^4 = 16$

$r = \pm 2$

$g_{10} \rightarrow$

$$\begin{array}{c|cc} n & 7 & 10 \\ \hline g_n & 12 & \end{array}$$

$\times r \ \times r \ \times r$ $\quad$ 3 more jumps of r

$g_{10} = 12r^3 =$

$g_{10} = 12(\pm 2)^3$

$= \pm 96$

b) Need the 1st term

$$\begin{array}{c|cc} n & 1 & 3 \\ \hline g_n & & \end{array}$$

go back 2 jumps of r so ÷ by r^2

$g_1 = \frac{\frac{3}{4}}{r^2}$

$g_1 = \frac{\frac{3}{4}}{(\pm 2)^2}$

$g_1 = \frac{\frac{3}{4}}{\cancel{\frac{4}{1}}} \cdot \frac{1}{4}$

$g_1 = \frac{3}{16}$

So: $g_n = g_1 \cdot r^{n-1}$

$3072 = \frac{3}{16}(\pm 2)^{n-1}$

$16384 = (\pm 2)^{n-1}$ ← Now take ln both sides

$\ln 16384 = (n-1)(\ln 2)$ ← bring exponent down

$n = \frac{\ln 16384}{\ln 2} + 1$ ← ÷ by ln 2 & add 1

$n = 15$

4. A rubber ball is dropped from a height of 10 m and bounces to reach $\frac{5}{6}$ of its previous height after each rebound. Let u_n is the ball's maximum height **before** its nth rebound.

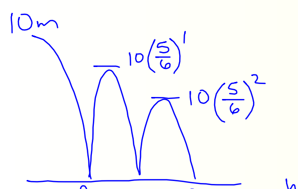
(a) Find an expression for u_n. $= 10\left(\frac{5}{6}\right)^{n-1}$

(b) How high will the ball bounce **after** its 5th rebound. so need u_6

(c) How many times has the ball bounced by the time it reaches a maximum height of $\frac{6250}{1296}$ m.

b) $u_6 = 10\left(\frac{5}{6}\right)^5$

≈ 4.02 m



u_n = ht before n^{th} rebound

n	1	2
u_n	10	$10\left(\frac{5}{6}\right)$

c) $\frac{6250}{1296} = 10\left(\frac{5}{6}\right)^{n-1}$

$\frac{625}{1296} = \left(\frac{5}{6}\right)^{n-1}$

$\frac{\ln\left(\frac{625}{1296}\right)}{\ln\frac{5}{6}} = \frac{(n-1)\ln\left(\frac{5}{6}\right)}{\ln\frac{5}{6}}$

$n = \frac{\ln\frac{625}{1296}}{\ln\frac{5}{6}} + 1$

$n = 5$

So the ball has bounced 4 times.

5. The terms $k+4$, $5k+4$, $k+20$ are in a geometric sequence. Find the value(s) of k.

Use the common ratios =

$$\frac{5k+4}{k+4} = \frac{k+20}{5k+4}$$

$$(5k+4)^2 = (k+4)(k+20)$$

$$25k^2 + 40k + 16 = k^2 + 24k + 80$$

$$24k^2 + 16k - 64 = 0$$

$$3k^2 + 2k - 8 = 0$$

$$(3k-4)(k+2) = 0$$

$$\boxed{k = \frac{4}{3}, -2}$$

6. A computer depreciates each year to 80% of its value from the previous year. When bought the computer was worth $8000.

(a) Find its value after

i. 3 years

ii. 6 years

(b) How long does it take for the computer to depreciate to a quarter of its purchase price.

$r = 0.8$

After 0 yrs.	After 1 yr.	2	After 3 yrs.
8,000			

3 jumps of r

ai) $8000r^3 = 8000(0.8)^3$

$= \$4096$

ii) After 6 yrs $= 8000(0.8)^6$

$\approx \$2,097.15$

b) $\frac{1}{4}(8000) = 8000(0.8)^t$ $\div$ by 8000

$\frac{1}{4} = (0.8)^t$

$\ln(0.25) = \ln(0.8)^t$

$\frac{\ln 0.25}{\ln 0.8} = \frac{t\,\cancel{\ln 0.8}}{\cancel{\ln 0.8}}$

$t \approx 6.2$ yrs.

7. The sum of the first and third terms of a geometric sequence is 40 while the sum of its second and fourth terms is 96. Find the sixth term of the sequence.

$g_n = g_1 \cdot r^{n-1}$

$g_1 + g_3 = 40$ $\qquad$ $g_2 + g_4 = 96$

$g_1 + g_1 r^2 = 40$ $\qquad$ $g_1 r + g_1 r^3 = 96$

① $g_1(1 + r^2) = 40$ $\qquad$ ② $g_1 r(1 + r^2) = 96$

Solve System $\left\{ \frac{②}{①} \right. \rightarrow$ $\frac{\cancel{g_1} r \cancel{(1+r^2)}}{\cancel{g_1}\cancel{(1+r^2)}} = \frac{96}{40}$

$r = \frac{12}{5}$ plug into eq. ①

① $g_1\left(1 + \left(\frac{12}{5}\right)^2\right) = 40$

$g_1\left(\frac{169}{25}\right) = 40$

$\boxed{g_1 = \frac{1000}{169}}$

The big finish:

$g_6 = \frac{1000}{169}\left(\frac{12}{5}\right)^5$

≈ 471

8. The sum of three successive terms of a geometric sequence is $\frac{35}{2}$ while their product is 125. Find the three terms.

$g_1 + g_2 + g_3 = \frac{35}{2}$

$g_1 + g_1 r + g_1 r^2 = \frac{35}{2}$

① $g_1(1 + r + r^2) = \frac{35}{2}$

$g_1 \cdot g_2 \cdot g_3 = 125$

$g_1 \cdot g_1 r \cdot g_1 r^2 = 125$

② $(g_1)^3 r^3 = 125$

$(g_1 r)^3 = 5^3$

$g_1 r = 5$

$r = \frac{5}{g_1}$

Sub into ①

Now solve system. Substitution this time

$g_1\left(1 + \frac{5}{g_1} + \frac{25}{(g_1)^2}\right) = \frac{35}{2}$

$2g_1\left(g_1 + 5 + \frac{25}{g_1}\right) = \frac{35}{2} \cdot 2g_1$

$2(g_1)^2 + 10g_1 + 50 = 35g_1$

$2(g_1)^2 - 25g_1 + 50 = 0$

$(2g_1 - 5)(g_1 - 10) = 0$

$\boxed{g_1 = \frac{5}{2}, 10}$

$r = \frac{5}{\frac{5}{2}}$ or $\frac{5}{10}$

$\boxed{r = 2 \text{ or } \frac{1}{2}}$

So the 3 terms

$\left\{\frac{5}{2}, 5, 10\right\}$

OR

$\left\{10, 5, \frac{5}{2}\right\}$

9. The population in a town of 40,000 increases at 3% per annum. Estimate the town's population after 10 years.

P = population

t = # of yrs.

$P = 40{,}000(1 + 0.03)^t$

$P(10) = 40{,}000(1.03)^{10}$

$\approx 53{,}757$

10. Following new government funding it is expected that the unemployed workforce will decrease by 1.2% per month. Initially there are 120,000 people unemployed. How large an unemployed workforce can the government expect to report in 8 months time.

$-1.2\% \rightarrow r = 1 - 0.012$

$r = 0.988$

U = unemployed

t = # of months

$U = 120{,}000(0.988)^t$

$U(8) = 120{,}000(0.988)^8$

$\approx 108{,}952$

So far:

Arithmetic $a_n = a_1 + d(n-1)$

$S_n = \frac{n}{2}(a_1 + a_n)$ or $\frac{n}{2}(2a_1 + d(n-1))$

Geometric $g_n = g_1 r^{(n-1)}$

Can we find the sum of a Geometric Series the same way we found the sum in an Arithmetic Series?

ex: 3+6+12+24+48+96

No

:(

$S_n = g_1 + g_1r + g_1r^2 + g_1r^3 + \ldots\ldots + g_1r^{n-3} + g_1r^{n-2} + g_1r^{n-1}$

$r\,S_n = \qquad g_1r + g_1r^2 + g_1r^3 + \ldots\ldots + g_1r^{n-3} + g_1r^{n-2} + g_1r^{n-1} + g_1r^n$

$$S_n - rS_n = g_1 - g_1r^n$$

$$S_n(1 - r) = g_1(1 - r^n)$$

$$\boxed{S_n = \frac{g_1(1-r^n)}{1-r}} \quad \text{or} \quad \frac{g_1(r^n-1)}{r-1}$$

Find: 3 + 6 + 12 + 24 + ... + 1536

Find: 3 + 6 + 12 + 24 + ... + 1536

first find n:

$$g_n = g_1 \cdot r^{n-1}$$

$$\frac{1536}{3} = \frac{\cancel{3}}{\cancel{3}}(2)^{n-1}$$

$$512 = 2^{n-1}$$

$$2^9 = 2^{n-1}$$

$$9 = n-1$$

$$n = 10$$

$$\boxed{S_n = \frac{g_1(1-r^n)}{1-r}}$$

$$S_{10} = \frac{3(1-2^{10})}{1-2}$$

$$S_{10} = -3(1-2^{10})$$

$$= 3069$$

Find the following sums:

$$\sum_{n=2}^{10} 3^n$$

$$\sum_{n=1}^{5} (-2)^n$$

$$\sum_{n=1}^{6} 3^{-n}$$

$$\sum_{n=1}^{8} 5(2)^{n-3}$$

Find the following sums:

$$\sum_{n=2}^{10} 3^n$$

$n = 10 - 2 + 1$

$n = 9$

$u_1 = 3^2 = 9$

$u_2 = 3^3 = 27$

$u_3 = 3^4 = 81$

$r = 3$

$$S_9 = \frac{9(1-3^9)}{1-3}$$

$$= \frac{9(1-3^9)}{-2}$$

$$= \boxed{88569}$$

$$\sum_{n=1}^{5} (-2)^n$$

$u_1 = (-2)^1 = -2$

$u_2 = (-2)^2 = 4$

$u_3 = (-2)^3 = -8$

$r = -2$

$$= \frac{-2(1-(-2)^5)}{1-(-2)}$$

$$= \frac{-2(1+32)}{3}$$

$$= \frac{-2(33)}{3}$$

$$= -2(11)$$

$$= \boxed{-22}$$

★ Notice the expressions are exponential & we can see what r is.

$$\sum_{n=1}^{6} 3^{-n} = \sum_{n=1}^{6} \left(\frac{1}{3}\right)^n$$

$$= S_6$$

$\frac{1}{3}, \frac{1}{9}, \frac{1}{27}$

$r = \frac{1}{3}$

$$\frac{\frac{1}{3}\left(1-\left(\frac{1}{3}\right)^6\right)}{1-\frac{1}{3}}$$

$$= \frac{\frac{1}{3}\left[1-\left(\frac{1}{3}\right)^6\right]}{\frac{2}{3}}$$

$$= \frac{3}{2}\cdot\frac{1}{3}\left[1-\left(\frac{1}{3}\right)^6\right]$$

$$= \frac{1}{2}\left[1-\left(\frac{1}{3}\right)^6\right]$$

$$\approx 0.499$$

$$\sum_{n=1}^{8} 5(2)^{n-3}$$

$$\boxed{\frac{1275}{4}}$$

To save the Eugene School District some money, I have decided to be paid 1 cent my first day, 2 cents on the second day, 4 cents on the third day, and so on in a geometric progression. How much will I get paid for my first week?

How much will I be paid for my first month (20 days) of work?

I work 190 days a year. What is my yearly salary?

I know the district is in a financial bind, so I agree to be paid for only my last day. How much would that be?

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$1 + 2 + 4 + 8 + 16 = 31¢$

How much will I be paid for my first month (20 days) of work?

$$S_{20} = \frac{1(1-2^{20})}{1-2}$$

$$= 1{,}048{,}575 ¢ \quad \text{or} \quad \$10{,}485.75$$

I work 190 days a year. What is my yearly salary?

$$S_{190} = \frac{1(1-2^{190})}{1-2} = 1.6\times10^{57} \text{ cents} = 1.6\times10^{55} \text{ dollars}$$

I know the district is in a financial bind, so I agree to be paid for only my last day. How much would that be?

$$g_{190} = 1\cdot 2^{189}$$

$$= 7.85\times10^{56} \text{ cents, OR } \$7.85\times10^{54}$$

HW: SL Book

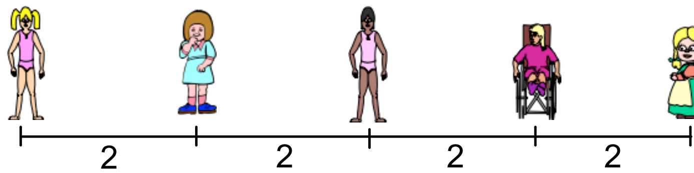
p. 259 #1d, 2c, 3ac,

4 -12, 14, 16

Actual
Quiz Tuesday:
8.1.1
8.1.2
8.1.3
8.2.1
8.2.2

Group Quiz Wednesday: Partial Fraction Decomposition

11. A children's game consists of the players standing in a line with a gap of 2 metres between each. The child at the left hand end of the line has a ball which s/he throws to the next child in the line, a distance of 2 metres. The ball is then thrown back to the first child who then throws the ball to the third child in the line, a distance of 4 metres. The ball is then returned to the first child, and so on until all the children have touched the ball at least once.



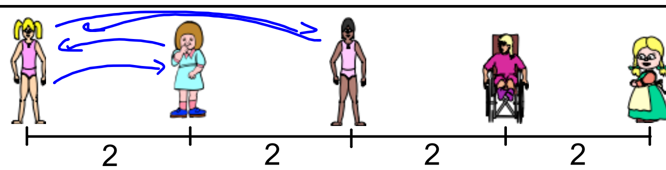
(a) If a total of five children play and they make the least number of throws so that only the leftmost child touches the ball more than once:

i. What is the largest single throw? 8

ii. What is the total distance travelled by the ball? $\boxed{40}$

let n = number of children playing

d_n = distance the ball travels back & forth between the 1st child and the n^{th} child. Make a table.



n	2	3	4	5
d_n	4 +	8 +	12 +	16
td	4	12	24	40

consider our sum formula:

$$S_n = \frac{n}{2}\left[2a_1 + d(n-1)\right]$$

$$S_4 = \frac{4}{2}\left[2(4) + 4(4-1)\right]$$

$$= 2(8+12)$$

$$= 40$$

when 5 children play we are on the 4th term in our sequence:

4, 8, 12, 16, . . .

(b) If seven children play, what is the total distance travelled by the ball? 84

(c) If n children play, derive a formula for the total distance travelled by the ball.
(d) Find the least number of children who need to play the game before the total distance travelled by the ball exceeds 100 metres.

c) $S_{n-1} = \frac{n-1}{2}\left[2(4) + 4(n-2)\right]$

Total $d = 2n(n-1)$
$= 2n^2 - 2n$

d) $d > 100$

(e) The children can all throw the ball 50 metres at most
i. What is the largest number of children that can play the game?
ii. What is the total distance travelled by the ball?

$2n^2 - 2n$ is distance travelled when n kids play

d. find the least number of kids who play before the total distance exceeds 100? Just set above equation > 100 and solve. The answer is n > 7.6 or -6.6. It can't be negative and must be an integer, so 8 kids play when distance exceeds 100

e. i. 25 gaps of 2 makes 50. 25 gaps means 26 kids

ii. Evaluate equation from part c when n = 26